Abstract—A synthetic aperture navigation (SAN) system that exploits downlink cellular long-term evolution (LTE) signals and an inertial measurement unit (IMU) is developed. The system is suitable for multipath-rich environments, such as indoors and deep urban canyons. The proposed SAN system mitigates multipath via a spatial discriminator, which utilizes the motion of a single antenna element to synthesize a geometrically separated antenna array from time-separated snapshots, alleviating the need for a physical antenna array. Signals from the synthesized antenna array are used to beamform towards the line-of-sight (LOS) LTE direction, while suppressing multipath components. Different stages of the beamforming process are discussed, and the computational complexity of the proposed system is analyzed. To deal with the unknown clock biases of the LTE eNodeBs, two navigation frameworks are developed: (1) base/rover and (2) standalone rover. The proposed SAN system is validated experimentally, and the navigation solutions achieved from the following systems are compared: (i) IMU only, (ii) LTE only, (iii) feedforward LTE-SAN, (iv) feedback LTE-SAN, (v) LTE-IMU, and (vi) feedback LTE-SAN-IMU. In the performed experiment, a pedestrian-mounted receiver navigated indoors for 109 min, while receiving LTE signals from 5 LTE eNodeBs. The proposed LTE-SAN-IMU system exhibited a two-dimensional position root mean-squared error (RMSE) of 1.44 m with a standard deviation of 1.85 m.

Index Terms—LTE, synthetic aperture, IMU, multipath mitigation, navigation, positioning, localization.

I. INTRODUCTION

People nowadays spend a tremendous amount of time in indoor structures, causing them to be known as the “indoor generation.” For example, Americans spend, on average, 90% of their time indoors. In light of this, demand for accurate indoor navigation and localization systems has been more than ever before. Not only accurate indoor navigation enables emerging applications, e.g., location-based services (LBS), it is vital for public safety, e.g., first responders. This has led the U.S. Federal Communication Commission (FCC), and its equivalent international counterparts, to pass mandatory requirements for indoor location accuracy on wireless devices [1]. However, even if these requirements are met, they will not provide sufficient accuracy for first responders and LBS applications. To motivate this, consider for example the Empire State building, a 102-story Art Deco skyscraper located in midtown Manhattan, New York city (see Fig. 1). New York city’s emergency 911 system handles more than 11 million calls per year, with at least 80% of the calls originating from wireless devices [1]. Imagine an enhanced 911 (E911) call originating from a wireless device within this building, to which emergency responders are dispatched. If met, the current FCC requirements (50-meter horizontal accuracy for 50% of all wireless calls) will only allow us to determine that the call is within this building. This is a rather large footprint for the responder to cover effectively and promptly, considering that the building is composed of 102 floors, each of which containing nearly 65 rooms. This example highlights how an accurate navigation system not only enhances public safety, but also, could save time and effort.

Numerous competing approaches have been proposed over the past couple of decades for indoor navigation and localization; however, there is no single technology that has emerged as a clear winner in solving this problem. Some of the most noteworthy approaches to date are summarized in Table I.

Among all approaches, cellular long-term evolution (LTE)-based approaches are particularly attractive as they are infrastructure-free, and if properly exploited, can lead to a practical, affordable, and accurate localization system. This is due to the inherent desirable characteristics of LTE signals [19], [20]: abundance, geometric diversity, high bandwidth (up to 20 MHz), high received power (carrier-to-noise ratio \( C/N_0 \)) ranges between 50 and 80 dB-Hz in different indoor conditions), and some of their downlink signals are free to use. Exploiting LTE-based signal for indoor localization comes with several challenges: (i) specialized receivers to opportunistically extract navigation observables from received LTE signals must be designed, (ii) the clock biases of LTE base stations (also known as evolved Node Bs or eNodeBs) must be removed or estimated, and (iii) errors resulting from short-delay multipath must be mitigated.

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Designing specialized LTE navigation receivers has been the topic of several studies over the past decade [21]–[26]. Meter- and sub-meter level accuracy was achieved outdoors using LTE signals on ground vehicles [15], [27]–[32] and aerial vehicles [33], respectively. However, the performance of these receivers degrades indoors, mainly due to short-delay multipath effects.

Multipath mitigation has been the subject of extensive studies in the literature, as it arises in different contexts. Some of the proposed techniques include: adaptive signal-to-noise ratio (SNR) [34], smart antennas [35], virtual multipath [36], multipath-estimating delay-locked loop (MEDLL) [37], cell-averaging constant false alarm rate (CA-CFAR) [29], sparsity-promoting regularization [38], and large-scale arrays [39].

In light of beamforming capabilities in existing and future systems (e.g., 5G), spatial discrimination offers an attractive approach to mitigate multipath by taking advantage of the geometric diversity of received signals. This can be done using physical antenna arrays [25], [40] or via synthetic aperture navigation (SAN) [17], [41]–[43]. Spatial discrimination techniques rely on the ability of beamforming towards the line-of-sight (LOS) direction while mitigating the multipath components. Originally termed in [44], SAN has been adopted for global navigation satellite (GNSS) signals [45]. In [46], an approach that utilizes the motion of an antenna element to enhance the detection of GNSS signals was presented, showing a 6 dB gain over a static receiver. The motion of an antenna array was exploited to minimize GNSS multipath errors in [42] and simulation results showed almost distortionless correlation peaks in the presence of one multipath signal when using a two-element antenna array. A preliminary study that applied SAN to LTE signals was conducted in [43] and [17]. The work in [47] improved the beamforming approaches presented in [43] and [17], where cascaded deep neural networks (DNN) were designed and trained using an LTE simulator in different environments to perform spatial smoothing (SS), model order estimation (MOE), and direction-of-arrival estimation (DOAE), which are three of the main stages in the beamforming process.

In signal-based navigation, NLOS scenarios can stand in the way of achieving accurate navigation performance. For decades, this problem has been the subject of significant research [9], [48]–[50]. Tackling this problem can be divided into two consecutive parts: (a) detecting the reception scenario and (b) addressing the detected scenario algorithmically. For (a), the studies pointed out three categories of wireless channels [48]: (i) dominant LOS, (ii) weak LOS, and (iii) NLOS. For a dominant LOS, the navigation system will perform just well. For a weak LOS, several approaches were proposed in the literature to address this challenge: (1) coherent/noncoherent integration of the incoming signal [51], which requires space (motion) or frequency diversity for multipath signals to de-correlate with time; (2) constant false rate alarm (CFAR) to enhance LOS reception [29]; and (3) advanced correlation detectors [52]. The proposed system mainly addresses this scenario by utilizing the motion of the receiver to spatially suppress the NLOS components and integrate the LOS component resulting in a dominant LOS scenario for the navigation receiver. For the NLOS scenario, conventional approaches eliminate the detected NLOS outliers from the measurement vector [53], [54]. The proposed approach drops the NLOS outliers, which are detected via a spatial discriminator of the LOS DOA. Another approach utilizes NLOS components as virtual transmitters and exploit them for navigation in a simultaneous localization and mapping (SLAM) fashion [21].

However, for detecting the reception scenario, two main approaches have been adopted in the literature: (i) LOS power discriminators and (ii) characterizing the delay profile of the signal components. The latter is more robust to signal power fluctuations; however, it requires estimating the multipath delays along with the LOS delay. For this purpose, the proposed system applies a spatial discriminator to detect NLOS outliers, in which the variation of the estimated LOS DOA is measured in vector fashion relative to previous estimates. For a pedestrian-type of motion, this approach showed a robust performance.

This paper develops an infrastructure-free navigation system that is suitable for multipath-rich environments, such as indoors and deep urban canyons. The developed system exploits downlink cellular LTE signals and an inertial measurement unit (IMU). The developed system couples navigation

<table>
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<tr>
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<th>Most notable results</th>
<th>Challenges</th>
</tr>
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<td>WiFi</td>
<td>[2]–[4]</td>
<td>80 percentile accuracy of 5.6 m [2].</td>
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$^1$Inertial navigation system, $^2$Root mean-squared error, $^3$Ultra-wideband, $^4$Radio-frequency identification, $^5$Circular error probability.

### TABLE I

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observables produced from received LTE signals with the IMU in a tightly coupled fashion. The navigation system mitigates multipath via a SAN-based spatial discriminator. This paper extends the work in [18] and [17] and makes the following contributions: (i) develop an extended Kalman filter (EKF)-based tightly-coupled LTE-SAN-aided IMU navigation system, (ii) study the complexity of the proposed system, and (iii) validate the proposed system experimentally using real LTE signals and an IMU.

Throughout the paper, italic small bold letters (e.g., \( x \)) represent vectors in time-domain, italic capital bold letters (e.g., \( X \)) represent vectors in frequency domain, and capital bold letter (e.g., \( X \)) represent matrices. The superscript \(^T\) denotes transpose symbol. The superscript * denotes the complex conjugate. The letters \( i \), \( k \), and \( n \) represent the symbol, subcarrier, and time index, respectively.

The remainder of the paper is organized as follows. Section III discusses: (i) the LTE received signal model, (ii) the LTE carrier phase-based receiver, (iii) IMU dead reckoning model, (iv) receiver clock state dynamics model, and (v) small-scale fading channel characterization. Section IV presents the proposed LTE-SAN approach and discuss: (i) the LTE-SAN model, (ii) spatial smoothing, (iii) model order estimation, (iv) DOA estimation using estimation of signal parameters via rotational invariance techniques (ESPRIT), and (v) multipath mitigation via Capon’s beamformer. Section V presents: (i) the base/rover framework to address eNodeBs’ clock biases challenge and (ii) the LTE-SAN-IMU tightly-coupling. Section VI assesses the computational complexity of the proposed system. Section VII validates experimentally the proposed LTE-SAN framework in an indoor environment and presents: (i) environmental layout and experimental setup, (ii) EKF initialization and settings, (iii) navigation solution, and (iv) SAN-based beamforming results. Concluding remarks are given in Section VIII.

II. OVERVIEW OF PROPOSED SYSTEM

This section presents a high-level block diagram of the proposed system. The proposed system could operate in one of two navigation frameworks: (i) base/rover or (ii) standalone rover frameworks. On one hand, let’s consider the base/rover framework. Imagine firefighters coming to a building, the fire truck will be outside the building. The truck is equipped with a GNSS antenna and LTE cellular antenna, which are connected to an RF front end to down-mix signals to baseband. The baseband in-phase and quadrature components of the mixer are fed to a stationary unit denoted by “base”. The base is nothing but an LTE carrier phase-based receiver that collects LTE signals from multiple carrier frequencies, which correspond to multiple LTE eNodeBs in the environment. The positions of the eNodeBs are pre-surveyed and assumed to be known. Moreover, the base is outdoors and has access to GNSS signals, so it can estimate its position. At the same time, the firefighters will step into the building equipped with a unit denoted by “rover.” The rover includes (i) an IMU and (ii) a copy of the same LTE receiver used in the base unit; however, this LTE receiver is integrated with an SAN correction block in which the motion of the firefighters is utilized to synthesize a geometrically-separated antenna array from time-separated snapshots. This allows for beamforming towards LOS while suppressing multipath components. The SAN correction block refines the carrier phase estimates and feeds, the refined estimate either in (a) a feedforward fashion to the navigation filter or in (b) a feedback fashion to the receiver. The IMU measurements are used to propagate the states of the rover. In the base/rover framework, the “known” ranges between the base and the eNodeBs are removed and the base measurements \( \{ \phi_{\text{base}}^{(u)} \}_{u=1}^U \) are subtracted from the corresponding rover measurements \( \{ \phi_{\text{rov}}^{(u)} \}_{u=1}^U \) to eliminate the eNodeBs’ clock biases, where \( U \) is the total number of eNodeBs. By eliminating the eNodeBs’ clock biases, the navigation filter estimates the 2-D position, velocity, orientation, and clock bias and drift of the rover. On the other hand, in the standalone rover framework, the base unit is not there and the rover estimates the difference between its own clock bias and drift and each eNodeB clock bias and drift. Fig. 2 presents an overview of the proposed system.

III. MODEL DESCRIPTION

This section presents the block diagram of the proposed system along with the various models adopted in the proposed indoor navigation system: (i) LTE signal model, (ii) LTE carrier phase-based receiver, (iii) IMU model, (iv) receiver clock state dynamics model, and (v) small-scale fading channel model.

A. LTE Signal Model

Downlink transmitted LTE signals deploy a multi-carrier modulation technique known as orthogonal frequency division multiplexing (OFDM), where all subcarrier signals within a communication channel are orthogonal to one another. The orthogonality allows for efficient modulation and demodulation implementation using the fast Fourier transform (FFT) algorithm on the receiver’s side, and inverse FFT (IFFT) on the transmitter’s side. OFDM is more resistant to intersymbol interference caused by multipath propagation due to: (i) low symbol rate modulation schemes (low-rate streams in parallel instead of a single high-rate stream) and (ii) inserted guard intervals between OFDM symbols. In the guard interval, a partial copy of the OFDM symbol, known as the cyclic prefix (CP), is transmitted so that the receiver integrates over integer number of sinusoid cycles for each multipath signal. Intersymbol interference can be avoided if the multipath time-spreading is shorter than the CP, which is 4.7 \( \mu \text{s} \) for LTE, when applying normal CP. This corresponds to a maximum difference of 1.4 km between the path length between transmitter and receiver. Two transmission modes are possible in LTE: time division duplexing (TDD) and frequency division duplexing (FDD). The majority of network providers use the FDD transmission mode for LTE due to its superior performance in terms of latency and transmission range. Therefore, this paper considers the LTE FDD transmission.

In LTE FDD transmitted data, the data streams are modulated onto multiple closely spaced carriers with a specific
carrier spacing $\Delta f = 15$ kHz, which is independent of the LTE system bandwidth that may take any of the following values: 1.4, 3, 5, 10, 15, and 20 MHz. Among these multiplexed data streams, several reference signals can be exploited from LTE signals and used for navigation purposes: (i) primary synchronization signal (PSS), (ii) secondary synchronization signal (SSS), (iii) positioning reference signal (PRS), and (iv) cell-specific reference signal (CRS). PSS and SSS are broadcasted by each eNodeB to provide frame start time and eNodeB’s cell ID to the user equipment (UE). Both PSS and SSS have a fixed bandwidth of 0.93 MHz, regardless of the variable LTE system bandwidth. The PRS was introduced in LTE release-9 to allow proper ranging measurements of the UE from LTE eNodeBs. CRS is designed for: (i) downlink channel estimation (for coherent demodulation and detection at the UE), (ii) cell search and initial acquisition, and (iii) downlink channel quality measurement. CRS has the same bandwidth as the LTE system bandwidth. Such higher bandwidth makes the CRS more capable to alleviate multipath-induced errors by providing high resolution in the time-domain. It is worth mentioning that all of the aforementioned signals can be used as reference signals to produce both pseudorange and carrier phase measurements. In this paper, CRS-based post-correlation data is used to perform beamforming algorithms on received LTE signals.

The channel frequency response (CFR) of the received CRS can be estimated, which is defined at the $i$-th symbol and $m$-th subcarrier as

$$\hat{H}_{i,m} = R_{i,m}S_{i,m}^*,$$  

where $R_{i,m}$ and $S_{i,m}$ are the received CRS and the receiver-generated CRS resource elements at the $i$-th symbol and $m$-th subcarrier, respectively; and $M$ is the number of CRS subcarriers, which varies depending on the LTE system bandwidth as shown in Table II. Further details on transmitted and received signal modeling of LTE signals can be found in [30].

### B. LTE Carrier Phase-Based Receiver

Several LTE receivers have been proposed to obtain pseudorange measurements from LTE signals [22, 29, 55, 56]. Analytical and experimental results have shown that the performance of these receivers significantly degrades in presence of multipath, making them unusable in indoor environments [16, 27, 30]. The LTE carrier phase-based receiver adopted in this paper, depicted in Fig. 3, is an adaptation of the receiver proposed in [30]. This receiver does not consider the code phase tracking loops utilized in the receiver developed in [30], since it was noticed that relying on carrier phase tracking loops alone was more robust in indoor environments [20]. However, the carrier phase-based receiver suffers from the need for ambiguity resolution. In the proposed approach, the ambiguity is resolved using an initial estimate of the code start time by: (i) having a prior position using GNSS signals or (ii) at the acquisition stage via estimating the initial code phase estimate of the received CRS signal. Although this approach does not provide high precision, it has low complexity compared to other ambiguity resolution techniques and can be performed on-the-fly.

<table>
<thead>
<tr>
<th>Bandwidth (MHz)</th>
<th>Total number of SCs</th>
<th>Number of used SCs</th>
<th>Number of CRS SCs $(M)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>128</td>
<td>72</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>256</td>
<td>180</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>512</td>
<td>300</td>
<td>50</td>
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<tr>
<td>10</td>
<td>1024</td>
<td>600</td>
<td>100</td>
</tr>
<tr>
<td>15</td>
<td>1536</td>
<td>900</td>
<td>150</td>
</tr>
<tr>
<td>20</td>
<td>2048</td>
<td>1200</td>
<td>200</td>
</tr>
</tbody>
</table>

SC: subcarrier.
C. IMU Model

The attitude of the UE is assumed to be obtained using an external sensor (e.g., a barometer). Therefore, only the two-dimensional (2-D) position \(G \mathbf{r}\), velocity \(\dot{G} \mathbf{r}\) in the global frame \(G\) and orientation with respect to the \(z\)-axis \(\theta_z\) are considered. The IMU produces the measurement vector \(z_{imu} = [\dot{\theta}_{zimu}, G \mathbf{r}_{imu}]^T\), where \(\dot{\theta}_{zimu}\) is the angular rate around the \(z\)-axis and \(G \dot{\mathbf{r}}\) is the specific force along \(x\)- and \(y\)-axes, which are modeled according to

\[
\begin{align*}
\dot{\theta}_{zimu}(n) &= \theta_z(n) + b_{\theta}(n) + n_{\theta}(n), \\
\dot{\mathbf{r}}_{imu}(n) &= \mathbf{R}(\theta_z(n)) \dot{G} \mathbf{r}(n) + b_{\mathbf{r}}(n) + n_{\mathbf{r}}(n),
\end{align*}
\]

where \(\dot{G} \mathbf{r}\) is the 2-D acceleration of the IMU in the global frame \(G\); \(\mathbf{R}(\theta_z) \in \mathbb{R}^{2 \times 2}\) is the rotation matrix representing the orientation of the body frame with respect to the global frame (as shown in Fig. 4) and is defined as

\[
\mathbf{R}(\theta_z) = \begin{bmatrix}
\cos \theta_z & -\sin \theta_z \\
\sin \theta_z & \cos \theta_z
\end{bmatrix};
\]

\(b_{\theta}\) represents the biases in the two accelerometers (\(x\)- and \(y\)-axes); \(n_{\theta}\) and \(n_{\mathbf{r}}\) are the gyroscopes’ and accelerometer’s measurement noise, which are modeled as zero-mean white noise sequences with covariances \(\sigma_{\theta}^2\) and \(\sigma_{\mathbf{r}}^2\), respectively. The evolutions of \(b_{\theta}\) and \(b_{\mathbf{r}}\) are modeled as random walk processes, i.e.,

\[
\begin{align*}
\dot{b}_{\theta}(n+1) &= b_{\theta}(n) + w_{\theta}(n), \\
\dot{b}_{\mathbf{r}}(n+1) &= b_{\mathbf{r}}(n) + w_{\mathbf{r}}(n),
\end{align*}
\]

with \(\mathbb{E}[w_{\theta}] = 0, \mathbb{E}[w_{\mathbf{r}}] = 0, \text{cov}[w_{\theta}] = \sigma_{\theta}^2, \text{and}\ \text{cov}[w_{\mathbf{r}}] = \sigma_{\mathbf{r}}^2\). The IMU measurements are used to evolve the position and orientation according to

\[
\begin{align*}
\dot{\theta}_z(n+1) &= \theta_z(n) + T \dot{\theta}_z(n), \\
G \dot{\mathbf{r}}(n+1) &= G \mathbf{r}(n) + T G \dot{\mathbf{r}}(n), \\
G \dot{\mathbf{r}}(n+1) &= G \dot{\mathbf{r}}(n) + T G \dot{\mathbf{r}}(n),
\end{align*}
\]

where \(T\) is the sampling interval.

D. Receiver Clock State Dynamics Model

The \(i\)-th receiver clock error state will be modeled as

\[
x_{clk,i}(n+1) = F_{clk} x_{clk,i}(n) + w_{clk,i}(n), \quad (6)
\]

for \(i \in \{\text{rov, base}\}\),

where \(x_{clk,i} \triangleq [\delta t_i, c \delta t_i]^T\) with \(\delta t\) and \(c\) being the clock bias and clock drift, respectively, \(c\) is the speed of light, \(F_{clk} \triangleq \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}\), and \(w_{clk,i}\) is modeled as a discrete-time zero-mean white random sequence with covariance \(Q_{clk,i}\) given by

\[
Q_{clk,i} = \begin{bmatrix} S_{\delta t,i} & T + S_{\delta t,i} \frac{T^2}{3} \\ S_{\delta t,i} \frac{T^2}{2} & S_{\delta t,i} T \end{bmatrix},
\]

where \(S_{\delta t,i}\) and \(S_{\delta t,i}\) are the power spectra of the continuous-time process noise \(w_{\delta t,i}\) and \(\tilde{w}_{\delta t,i}\) driving the clock bias and clock drift, respectively. The values of \(S_{\delta t,i}\) and \(S_{\delta t,i}\) depend on the clock’s quality [57].

E. LTE Carrier Phase Measurement Model

The carrier phase measurement, expressed in meters, made by the UE (base or rover) on the \(u\)-th LTE eNodeB can be shown to be

\[
\phi_{i,u}(n) \triangleq \phi_{i,u}(n) = \|r_i(n) - r_{su}(n)\|_2 + c[\delta t_i(n) - \delta t_{su}(n)] + b_{ui} + n_{ui}(n),
\]

for \(i \in \{\text{rov, base}\}\),

where \(\lambda_u\) is the wavelength of the transmitted signal, \(\psi_{i,u}\) is the carrier phase estimate produced by the receiver, \(r_i\) is the UE’s position, \(r_{su}\) is the \(u\)-th eNodeB’s position, \(\delta t_i\) is the UE’s clock bias, \(\delta t_{su}\) is the \(u\)-th eNodeB’s clock bias, \(b_{ui}\) is the \(u\)-th eNodeB’s initial carrier phase ambiguity expressed in meters, and \(n_{ui}\) is the \(u\)-th eNodeB’s measurement noise modeled as a zero-mean white sequence with variance \(\sigma_{\phi}^2\). The measurement noise for all eNodeBs is assumed to be independent.

The UE’s position is assumed to be known at \(n = 0\). Then, a new measurement is defined as

\[
\phi_{i,u}'(n) = \phi_{i,u}(n) - \phi_{i,u}(0) + \|r_i(n) - r_{su}\|_2
\]

\[
= \|r_i(n) - r_{su}\|_2 + c[\delta t_i(n) - \delta t_{su}(n)] + n_{ui}(n),
\]

where \(\delta t_i'(n) \triangleq (\delta t_i(n) - \delta t_{ui}(0))\) and \(\delta t_{su}(n) \triangleq (\delta t_{su}(n) - \delta t_{su}(0))\), and \(n_{ui}(n) \triangleq n_{ui}(n) - n_{ui}(0)\), where \(n_{ui}\) is modeled as \(n_{ui}\) with variance \(\sigma_{\phi}^2\). For simplicity, the “′” is dropped for the rest of the paper.
F. Small-Scale Fading

The LTE received signals will be distorted due to small-scale fading if the bandwidth of transmitted signal is much greater than the coherence bandwidth $B_c$. The coherence bandwidth helps in defining the boundary between narrowband and wideband signals as the maximum frequency bandwidth for which the signals are still considered correlated (e.g., coherence coefficient $\geq 0.5$). Multipath channels are usually characterized by the power delay profile, which shows the received power versus time. The power profile could be treated as a non-normalized probability density function from which mean delay, mean-squared delay, and consequently the root mean-squared delay $\tau_{rms}$ spread can be computed. This delay and the coherence bandwidth are inversely related via the rule of thumb as [58]

$$\frac{1}{5\tau_{rms}} < B_c < \frac{1}{5\tau_{rms}}.$$ 

For indoor environments, experimental results showed that $10 \text{ ns} \leq \tau_{rms} \leq 50 \text{ ns}$ [59].

The Doppler spread along with coherence time provides information about the time-varying characteristic of the wireless channel. The Doppler spread $B_D$ is a measure of the spectral broadening caused by the time rate of change of mobile radio channel and it specifies the spectrum where the received Doppler is non-zero. For terrestrial signal-based application, the dynamics of the receiver determines $B_D$. If the incoming signal bandwidth is significantly greater than $B_D$, then the Doppler spread effect is negligible. The coherence time $T_c$ is inversely proportional to $B_D$ and it characterizes the frequency dispersive nature of a wireless channel in the time-domain. It represents the maximum time for which the signals are still considered to be correlated. It helps define the boundary between slow and fast fading signals. A rule of thumb to calculate $T_c$ is

$$T_c \approx \frac{0.423}{f_{D,\text{max}}},$$

where $f_{D,\text{max}}$ is the maximum Doppler frequency introduced by the motion of the receiver, which can be calculated based on the maximum speed $v_{\text{max}}$ of the receiver as $f_{D,\text{max}} = v_{\text{max}}/\lambda$, where $\lambda$ is the wavelength of the transmitted signal.

IV. Synthetic Aperture Navigation with LTE Signals

This section develops a spatial-time discriminator for navigation using LTE signals in multipath environment. The proposed approach discriminates the LOS from the multipath components using spatial information extracted from the received LTE signals. The spatial separation is performed using a synthetic aperture antenna array, where a moving single antenna element synthesizes an antenna array with time-separated elements. The proposed system assumes a synthetic aperture uniform linear array (ULA) of $N$ time-separated antenna elements, which corresponds to $N$ LTE snapshots (symbols) captured by the moving antenna and each separated by $F$ symbols, where the OFDM symbol duration is $t_{\text{symb}} = 66.7 \mu s$. In practice, the UE’s trajectory could be arbitrary with a time-varying speed $v(n)$, as shown in Fig. 5. However, the proposed approach assumes $v(n) = v_c$ to be constant over a short duration of time denoted as the synthetic time $T_S = N F t_{\text{symb}}$ seconds. This assumption is reasonable for typical pedestrian indoor motion, where the speed of a pedestrian indoors ranges from 0.5 m/s up to 2.5 m/s. The pedestrian speed $v_c$ could be either estimated in the navigation filter or measured via an external sensor, e.g., IMU. Inhere, $v_c$ is estimated in the navigation filter by taking the average over $T_S$. The skipped OFDM symbols $F$ accounts for the physical separation between two consecutive snapshots of the moving antenna, i.e., $F$ is determined as $F = \lfloor \frac{d}{t_{\text{symb}} v_c} \rfloor$, where $d$ is the spacing between two consecutive synthesized antenna elements and $\lfloor \cdot \rfloor$ rounds the argument to the nearest integer. For design purposes, $d$ is defined as $d \equiv \alpha \lambda$, where $\alpha$ is the antenna spacing ratio to be chosen adaptively when building the synthetic antenna array.

When choosing the parameter $N$, few points need to be taken into consideration: (1) the stationarity of the LTE wireless channel, which differs from one environment to another and relates to the Doppler spread as discussed in Subsection III-F, (2) the desired degree-of-freedom (DOF) (i.e., the number of received multipath signals that could be estimated effectively), and (3) the computational cost of the DOA estimation, which increases exponentially with $N$. The altitude of the receiver is assumed to be obtained using an external sensor (e.g., a barometer); i.e., only the azimuth angles $\{\phi^{(u)}\}_{l=0}^{L^{(u)}}$ of the impinging signals (both LOS and multipath signals) from the $u$-th eNodeB are estimated, where $L^{(u)}$ is the number of multipath components. Fig. 5 depicts the proposed LTE-SAN approach with a synthetic ULA. The following subsections formulate the LTE-SAN framework, discuss DOA estimation using ESPRIT subspace-based estimator, present the NLOS detector, and analyze the beamforming process to mitigate multipath.

Fig. 5. Synthetic ULA: UE trajectory, sampling process of the moving antenna, and a snapshot of the azimuth angle impinging on the antenna from the $u$-th eNodeB at instant $n$.

The proposed approach uses an ad-hoc method to choose the antenna separation, where $d$ is chosen to satisfy the relationship $T_S \leq T_c$. Previous studies about DOA estimation have studied the optimal performance of spatial discrimination, which was achieved for spacing different from $\frac{d}{T}$ [60]–[62]. The optimal choice was shown to depend on various factors such as: type of signal, environment, and wavelength. In [61], where short interelement separation was utilized for CDMA signals indoors, the spatial envelope correlation was...
studied as a function of the antenna separation. The study showed how the spatial envelope correlation increases for $d < \frac{\lambda}{2}$, which decreases the DOA estimation accuracy. In the proposed system, spatial smoothing is applied to address this challenge by pre-filtering the incoming data to improve geometric diversity. This is achieved by dividing the synthetic antenna array into symmetric sub-antenna arrays; hence, the DOF of the system decreases, i.e., resolving fewer incoming signals. However, it is worth mentioning that choosing $d = \alpha \lambda$ has to consider the trade-off between geometric diversity, spatial correlation, and degree-of-freedom (DOF) of the spatial discriminator, while maintaining an acceptable error margin in DOA estimation.

The purpose is to design the synthetic antenna array that guarantees an acceptable DOA estimation accuracy and maximizes the size of synthetic array DOF while satisfying $T_S \leq T_C$. The proposed approach studied the DOA estimation accuracy as a function of the spacing in a Monte Carlo fashion. This study used the LTE simulator developed in [47] to simulate LTE signals assuming multipath-rich indoor environment. The study evaluated the DOA RMSE of the proposed system versus $\alpha \in [0 : 1]$ for four different LTE carrier frequencies. Fig. 6 shows the results. It can be seen how the performance is comparable/acceptable for $\alpha \in [0.05 : 0.5]$. For $\alpha < 0.05$, as $\alpha$ approaches zero, the spatial diversity approaches zero and the DOA RMSE increases exponentially. For $\alpha > 0.5$, a spatial ambiguity arises and the DOA RMSE grows logarithmically. Given these results, a lower bound $\alpha$, denoted $\alpha_{\text{min}}$, can be chosen to guarantee a specific DOA estimation accuracy. For instance, an accuracy of $10^\circ$ was considered in this application, which results in $\alpha_{\text{min}} \triangleq 0.05$ and $d_{\text{min}} = \alpha_{\text{min}} \lambda$. Then, the skipped LTE symbols between two consecutive snapshots and the maximum size of the synthetic array (maximum DOF) are obtained as

$$F = \left\lceil \frac{d_{\text{min}}}{\text{f}_{\text{symb}} v_c} \right\rceil \quad (8)$$

$$N \triangleq N_{\text{max}} = \left\lceil \frac{T_C}{F \text{f}_{\text{symb}}} \right\rceil, \quad (9)$$

where $\lceil \cdot \rceil$ denotes the greatest integer less than the argument.

A. LTE-SAN Signal Formulation

The LTE carrier phase measurements are produced by tracking the CRS in the received LTE signals using the carrier phase-based receiver presented in Subsection III-B. The CFR of the $n$-th received LTE frame is denoted as $H(n)$. The CFR is attractive due to high bandwidth of CRS. The proposed approach starts at the receiver’s post-correlation phase. The estimated CFR of the received LTE signal from the $u$-th at the $n$-th snapshot can be expressed as

$$H(n) = \sum_{l=0}^{L(u)} \alpha_l^{(u)} \alpha_l^{(u)} e^{-j2\pi f_c(\tau_l^{(u)} + nt_{\text{symb}}/N)}, \quad (10)$$

for $n = 1, \cdots, N$,

where $\alpha_l^{(u)}$ and $\tau_l^{(u)}$ are the attenuation factor and the delay, respectively, of the $l$-th multipath component, $f_c$ is the carrier frequency, and $a_l^{(u)}$ and $a_0^{(u)}$ are the steering elements of the antenna element at the $n$-th snapshot of the $l$-th multipath and LOS components, respectively. The steering element of a signal represents the phase delay a plane wave experiences, evaluated at the specified antenna element. The phases are specified with respect to an arbitrary origin, where in most DOA approaches it is chosen to be the axis of the antenna array centered at the first snapshot. For instance, an $n$-th element having a position of $r_n = (x_n, y_n)^T$, the steering element for this specific element is calculated as

$$a_l^{(u)}(r_n) = e^{-j \langle k, r_n \rangle},$$

where $k$ is the wave vector that describes the phase variation of a plane wave and $(\mathbf{a}, \mathbf{b})$ denotes the dot-product of vectors $\mathbf{a}$ and $\mathbf{b}$. The LOS steering element of the $u$-th eNodeB at the $n$-th snapshot, assuming knowledge of the eNodeB’s position, can be obtained as

$$a_0^{(u)}(r_n) = e^{(n-1)\mu^{(u)}},$$

where $\mu^{(u)} = -\frac{2\pi d}{\lambda} \sin(\phi_{\text{r}}^u)$ is the spatial frequency for the $u$-th eNodeB [40]. To simplify notation, the superscript “(u)” will be dropped for the rest of the paper. The complex representation of the $u$-th eNodeB CFR at time instance $n$ for a single snapshot can be expressed as

$$H(n) = \mathbf{a}(\phi_n) \mathbf{x}(n) + \nu(n), \quad (11)$$

where $\nu$ represents noise modeled as zero-mean complex white Gaussian with covariance $\sigma^2$ and

$$\mathbf{a}(\phi_n) = [a_0(\phi_n), \cdots, a_L(\phi_n)]^T,$$

$$\mathbf{x}(n) = [\alpha_0 e^{-j2\pi f_c(\tau_0 + nt_{\text{symb}}/N)), \cdots, \alpha_L e^{-j2\pi f_c(\tau_L + nt_{\text{symb}}/N)}]^T.$$ The signals captured from $N$ snapshots, each separated by $F$ OFDM symbols are combined as

$$\mathbf{H}(n) = [H(n), H(n + F), \cdots, H(n + (N - 1)F)]^T. \quad (12)$$

For $M$ samples, the collected data is stacked as

$$\mathbf{H}^{(N)} = [\mathbf{H}(n), \mathbf{H}(n + 1), \cdots, \mathbf{H}(n + (M - 1))]^T. \quad (13)$$
B. Preprocess Filtering, Model Order Estimation, and DOA Estimation

Different DOA estimation techniques exist in the literature, with different performance, resolution, and computational cost. Subspace-based DOA estimation techniques have better resolution than maximum-likelihood (ML) techniques. Subspace techniques basically rely on the fact that the spatial covariance matrix (i.e., signals plus noise) of the received data spans two orthogonal subspaces, namely, the signal and noise subspaces, where the signal subspace is spanned by the larger eigenvalues of the data covariance matrix. Multiple Signal Classification (MUSIC) is one of the most popular and early proposed methods for super-resolution DOA estimation [63]. When applied to LTE DOA estimation, MUSIC has yielded high-resolution performance [17]. However, MUSIC has a high computational cost. An alternative technique with a low computational cost and high-resolution capabilities is the ESPRIT algorithm. ESPRIT requires a symmetric geometric pattern and an invariance transformation characteristic for the applied antenna design. To do so, ESPRIT divides the array into two symmetric subarrays, which can be implemented in several ways, three of which are shown in Fig. 7. The subarrays in the red and blue boxes are defined by connection matrices denoted by $J_1$ and $J_2$, respectively. For linear arrays, the design shown in Fig. 7(a) provides the highest DOF (i.e., the DOF of the new subarray) and is adopted in the proposed approach.

In practice, the spatial covariance matrix $R_{HH}$ of the received LTE data in (13) is not known; however, an estimate of $R_{HH}$ could be obtained as

$$
\hat{R}_{HH} = \frac{1}{M} H^{(N)} H^{(N)\dagger}, \tag{14}
$$

where $(.)^\dagger$ is the Hermitian operator. Algorithm 1 summarizes the steps of the standard ESPRIT algorithm. Further details about ESPRIT can be found in [64].

1) Preprocessing Scheme: Spatial Smoothing: The DOA estimation algorithms described so far assume that the incoming signals are noncoherent. In other words, they assume that the steering matrix is full rank. If the impinging signals are highly correlated or coherent, different DOA estimation techniques will fail to provide reliable DOA estimates due to having an ill-conditioned or even singular spatial covariance matrix. In practical multipath scenarios, having highly correlated signals is very common where the incoming signals are scaled and delayed versions of each other. To overcome this challenge, the data covariance matrix is preprocessed before "feeding" it to the DOA estimation algorithm. Two well-known preprocessing schemes deal with this challenge: (1) forward-backward (FB)-averaging and (2) spatial smoothing [40]. FB-averaging is capable of resolving only the case of two coherent signals [65]. In rich multipath areas, the data may encounter more than two coherent signals. This raises the need for a more sophisticated approach to resolve this challenge. To this end, SS seems to be an attractive technique to tackle this issue. SS divides the antenna array into a smaller number of subarrays (denoted $C$) and the data covariance matrices obtained from each subarray are averaged. For 1-D SS, the ULA is divided into $N_{\text{sub}} = N - C + 1$ subarrays to decouple the eigenvectors of at most $C$ coherent signals, i.e., the data is expressed as

$$
H_s^{(N)} = \left[ H_{fss}^{(N)}, H_{bss}^{(N)} \right],
$$

where $H_{fss}^{(N)}$ and $H_{bss}^{(N)}$ are the forward and the backward spatially smoothed data, defined as

$$
H_{fss}^{(N)} = \left[ J_{f1} H_s^{(N)} J_{f2} H_s^{(N)} \ldots J_{fC} H_s^{(N)} \right],
$$

$$
H_{bss}^{(N)} = \left[ J_{b1} H_s^{(N)} J_{b2} H_s^{(N)} \ldots J_{bC} H_s^{(N)} \right],
$$

and

$$
J_{fc} = \begin{bmatrix} 0_{N_{\text{sub}} \times C} & I_{N_{\text{sub}}} & 0_{N_{\text{sub}} \times (N - N_{\text{sub}} - C + 1)} \end{bmatrix},
$$

$$
J_{bc} = \begin{bmatrix} 0_{N_{\text{sub}} \times (N - N_{\text{sub}} - C + 1)} & I_{N_{\text{sub}}} & 0_{N_{\text{sub}} \times C} \end{bmatrix}
$$

for $c = 1, \ldots, C$.
matrices are obtained as
\[
\begin{align*}
R_{HH}^{fss} &= \frac{1}{CM} H_{fss}^{(N)} H_{fss}^{(N)H}, \\
R_{HH}^{bs} &= \frac{1}{CM} H_{bs}^{(N)} H_{bs}^{(N)H}.
\end{align*}
\]
Finally, the overall spatially smoothed data covariance matrix
is obtained by averaging both the forward and backward subarrays as
\[
\hat{R}_{HH}^{ss} = \frac{1}{2} \left( \hat{R}_{HH}^{fss} + \hat{R}_{HH}^{bs} \right).
\]
Note that there is a trade-off between the number of coherent
signals to be resolved and the new DOF associated with the
new subarray’s size.

2) Model Order Estimators: In addition to the coherence
issue of incoming signals, the number of signals \( L + 1 \)
imponging on the array was assumed to be known so far.
 Practically, this number is unknown and has to be estimated
from the received data. The simplest way for estimating the
number of signals is by estimating the number of repeated
small eigenvalues other than the large ones. In other words,
if the multiplicity \( \hat{q} \) of the smallest eigenvalues is found, an
estimate of the number of signals, \( \hat{L} + 1 \), can be obtained
directly as
\[
\hat{L} = N - \hat{q} - 1.
\]
In practice, the smallest eigenvalues representing the noise
power will not be identical. Instead, they will appear as a
closely spaced cluster. This could be formed as a detection
problem where the number of incoming signals obtained by
a ULA is \( L \in \{0, 1, \cdots, N-1\} \). To estimate the order of
the system, one can apply the minimum description length
criterion (MDL) or Akaike information theoretic criterion
(AIC) [66], [67]. The estimate \( \hat{L} = \arg \min_J J \), where
\[
J = \begin{cases}
- \ln \left( \frac{\prod_{l=1}^{\hat{q}+1} \lambda_{l}^{(N/2)}}{\prod_{l=\hat{q}+2}^{N/2} \lambda_{l}} \right)^{(N-L-1)M} + \hat{p}_M, & \text{MDL} \\
- \ln \left( \frac{\prod_{l=1}^{\hat{q}+1} \lambda_{l}^{(N/2)}}{\prod_{l=\hat{q}+2}^{N} \lambda_{l}} \right)^{(N-L-1)M} + \hat{p}_A, & \text{AIC},
\end{cases}
\]
where \( \hat{p}_M \) and \( \hat{p}_A \) are functions of number of independent
parameters called penalty functions [68].

C. Multipath Mitigation: Capon’s Beamformer

In order to suppress multipath signals, the only signal that
is allowed to pass through the beamformer is the LOS signal.
After applying beamforming to the synthetic data, the data
received by array elements form a single output as follows
\[
y(n) = w^H H^{(N)},
\]
where \( w \) is a weighting vector that is determined by opti-
mizing some objective function subject to certain constraints.
Beamforming methods differ via the choice of their objective
functions and constraints. The common strategy behind beam-
forming is to steer the antenna array in a particular direction at
a time and evaluate the objective function, seeking an optimal
complex weighting vector to weight the received signals at
different snapshots. Herein, the purpose behind the proposed
LTE-SAN framework is to suppress the multipath components
while passing the beam where the LOS component impinges
on the synthetic antenna array. To do so, different beamform-
ing techniques could be applied. A potential beamforming
 technique is the Capon’s method, also known as minimum
variance distortionless response (MVDR) beamformer [69],
[70]. The chosen weighting vector for MVDR minimizes the
variance of the array output signal while passing the signal
arriving from the direction of interest with no distortion which
can be shown to be
\[
w = \frac{\hat{R}_{HH}^{-1} a_0(\phi)}{a_0^H(\phi) \hat{R}_{HH}^{-1} a_0(\phi)}. \tag{16}
\]
where \( a_0 \) is the LOS steering vector, which is obtained by
taking the nearest DOA estimate \( \{\phi_i\}_{i=1}^{L+1} \) from Algorithm 1
 to the LOS DOA estimate calculated using the current estimate
of the rover’s receiver and the known LTE eNodeB’s position.

V. LTE-SAN-IMU NAVIGATION FRAMEWORK

This section presents two navigation frameworks: (i) a
base/rover framework and (ii) a standalone UE framework.
In both frameworks, LTE navigation observables are fed to
the SAN-based beamformer to correct for multipath-induced
errors; then, they are tightly coupled with IMU measurements
using EKF.

A. Navigation Frameworks

One of the main challenges for navigation with LTE signals
is the unknown clock biases of LTE eNodeBs by the receiver,
which need to be either eliminated in a differential manner
[71], [72] or estimated [73], [74]. This challenge can be
overcome via either of the following frameworks:

1) Base/Rover Framework: Fig. 2 depicts this framework
in which a base is placed outdoors. The base can be mounted
on a fire truck or a police car, and can estimate its own position
from GNSS signals.

The base and the rover are assumed to receive signals
from the same LTE eNodeBs in the environment, located at
\( \{r_{s,u}\}_{u=1}^{U} \), where \( U \) is the total number of eNodeBs. The base
transmits its carrier phase measurements \( \phi_{base}^{(u)} \) to the rover,
which subtracts it from its own \( \phi_{rov}^{(u)} \) and adds the known
range between the base and the \( u \)-th eNodeB \( \| r_{base} - r_{s,u} \|_2 \)
to produce the measurement
\[
z_u = \phi_{rov}^{(u)} - \phi_{base}^{(u)} + \| r_{base} - r_{s,u} \|_2.
\]
Note that eNodeB clock bias is eliminated from \( z_u \). The
resulting measurements \( z \triangleq [z_1, \cdots, z_U]^T \) are fed to the EKF.

2) Standalone Framework: This framework consists of a
standalone rover, which estimated the difference between
its own clock bias and drift and each eNodeB clock bias and
drift, i.e., \( [\delta t_{rov} - \delta t_{s,1}]_{u=1}^{U} \) and \( [\delta t_{rov} - \delta t_{s,1}]_{u=1}^{U} \),
respectively. In this framework, the measurement vector that
is fed to the navigation filter is \( z \triangleq [\phi_{rov}^{(1)}, \cdots, \phi_{rov}^{(U)}]^T \).
B. LTE-SAN-IMU Coupling

The rover UE is assumed to be equipped with an IMU. An EKF is used to fuse the IMU measurements with \( z \) in a tightly-coupled fashion as shown in Fig. 8. A barometer can be used to estimate the rover’s altitude. Therefore, the EKF will only consider estimating the 2-D position of the rover.

1) LTE-SAN Coupling: The proposed SAN method discussed in Section IV beamforms the post correlation data and suppresses the effect of multipath signals to obtain new CIRs \( \{ y(k)\}^U_{u=1} \) with a dominant LOS peak. The new CIRs obtained are used to produce the corrected carrier phase measurements, denoted as \( \hat{z} \), to replace the old observables \( z \). This is achieved by coupling the LTE receiver with the SAN approach presented in Section IV. The block diagram in Fig. 8 presents two coupling schemes: (1) feedforward LTE-SAN and (2) feedback LTE-SAN.

In the feedforward LTE-SAN coupling scheme, where nodes A and B are connected to node 1, the measurements \( z \) generated by the LTE carrier phase-based receiver are processed in the proposed SAN algorithm. The corrected measurements \( \hat{z} \) are fused via an EKF with IMU data. In the feedback LTE-SAN coupling scheme, where nodes A and B are connected to node 2, the measurements \( z \) generated by the LTE carrier phase-based receiver are processed in the proposed SAN algorithm. The CIR is estimated in the tracking loop of the LTE receiver at each time instance. In the feedback scheme, the corrected CIR \( y(k) \) obtained using the proposed approach is fed back to the tracking loops and replaces the CIR estimated using the standalone LTE receiver, which is used to produce measurements \( \hat{z} \). Then, the corrected measurements \( \hat{z} \) are fused via an EKF with IMU data. Note that in Fig. 8, \( n \) and \( j \) are discrete-time instances where \( k > j \).

![Fig. 8. Block diagram of coupling LTE-SAN-IMU system.](image)

2) EKF State: The rover’s state vector \( x \) is defined as

\[
x = \begin{bmatrix} x_{\text{IMU}}^T, \, x_{\text{clk}}^T \end{bmatrix}^T,
\]

where \( x_{\text{IMU}} \) and \( x_{\text{clk}} \) are the IMU and clock state vectors, respectively.

The IMU state vector is defined as

\[
x_{\text{IMU}} \triangleq \left[ \theta, \, G, \, \dot{G}, \, B_{\alpha}, \, B_{\gamma} \right]^T.
\]

In the base/rover framework, the clock state vector is defined as

\[
x_{\text{clk}} = n_{\text{clk}} - n_{\text{clkbase}} = \begin{bmatrix} c(\delta t_{\text{rover}} - \delta t_{\text{base}}), \, c(\delta t_{\text{rover}} - \delta t_{\text{base}}) \end{bmatrix}^T,
\]

where \( \delta t_{\text{rover}} \) and \( \delta t_{\text{base}} \) are the clock biases of the rover and base receivers, respectively; and \( \delta t_{\text{rover}} \) and \( \delta t_{\text{base}} \) are clock drifts of the rover and base receivers, respectively.

In the standalone rover framework, the clock state vector is defined as

\[
x_{\text{clk}} \triangleq \begin{bmatrix} c(\delta t_{\text{rover}} - \delta t_{\text{base}}), \, c(\delta t_{\text{rover}} - \delta t_{\text{base}}) \end{bmatrix}^T.
\]

3) EKF Time Update: At time step \( n \), the EKF produces an estimate of the state vector \( \hat{x}(n|j) \) along with an estimation error covariance \( P(n|j) \), where \( n \geq j \); \( \mathbf{Z}^j = \{ z(l) \}_{l=1}^j \); and \( \tilde{x}(n|j) \) is the observation error. The EKF time update of the clock state estimate is given by

\[
\hat{x}_{\text{clk}}(n+1|j) = F_{\text{clk}} \hat{x}_{\text{clk}}(n|j).
\]

The prediction error covariance matrix is given by

\[
P(n+1|j) = FP(n|j)F^T + Q_d,
\]

where \( F = \text{diag} [F_{\text{IMU}}, F_{\text{clk}}] \); \( Q_d = \text{diag} [Q_{\text{IMU}}, Q_{\text{clk}}] \); \( F_{\text{IMU}} \) is the linearized discrete-time IMU state transition matrix given by

\[
F_{\text{IMU}} = \begin{bmatrix} 1 & 0_{1 \times 2} & 0_{1 \times 2} & 0_{1 \times 2} & T \\ \mathbf{I}(n|j) & I_{2 \times 2} & 0_{2 \times 2} & TR(\hat{\theta}(n|j)) & 0_{2 \times 1} \\ 0_{2 \times 1} & T T_{2 \times 2} & I_{2 \times 2} & 0_{2 \times 2} & 0_{2 \times 2} \\ 0_{2 \times 1} & 0_{2 \times 2} & 0_{2 \times 2} & I_{2 \times 2} & 0_{2 \times 1} \\ 0 & 0_{1 \times 2} & 0_{1 \times 2} & 0_{1 \times 2} & 1 \end{bmatrix},
\]

\[
\hat{\theta}(n|j) \triangleq J \mathbf{R}[\hat{\theta}(n|j)] (\hat{\mathbf{r}}_{\text{IMU}}(n|j) + \hat{b}_a(n|j)),
\]

with \( J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \); \( Q_{\text{IMU}} \) is the linearized discrete-time IMU state process noise covariance matrix given by

\[
Q_{\text{IMU}} = \frac{T}{2} F_{\text{IMU}} \mathbf{N}_e F_{\text{IMU}} + \mathbf{N}_e,
\]

where \( \mathbf{N}_e \triangleq \mathbf{G} Q \mathbf{G}^T; \mathbf{Q} \) is the continuous-time IMU process noise covariance matrix defined as

\[
\mathbf{Q} = \text{diag} \left[ \sigma^2_{\text{clk}}, \sigma^2_{\alpha}, 1, \sigma^2_{\omega_\alpha}, 1, \sigma^2_{\omega_\gamma}, 1 \right],
\]

and \( \mathbf{G} \) is the error-state transition matrix defined as

\[
\mathbf{G} = \begin{bmatrix} 1 & 0_{1 \times 2} & 0_{1 \times 3} \\ 0_{2 \times 1} & R(\hat{\theta}(n|j)) & 0_{2 \times 3} \\ 0_{2 \times 1} & 0_{2 \times 2} & 0_{2 \times 3} \\ 0_{3 \times 1} & 0_{3 \times 2} & I_{3 \times 3} \end{bmatrix}.
\]
4) EKF Measurement Update: Once the EKF receives the measurement vector $z'$, it performs a measurement update according to

$$
\hat{x}(n+1|n+1) = \hat{x}(n+1|j) + K(n+1)\nu(n+1),
$$

where $\nu$ and $K$ are the innovation vector and Kalman gain, respectively, given by

$$
\nu \triangleq z' - \hat{z}',
$$

$$
\hat{z}_u \triangleq \| \hat{r}(n+1|j) - r_{nu} \|_2 + c\Delta t(n+1|j),
$$

$$
K(n+1) \triangleq P(n+1|j)H^T(n+1)S^{-1}(n+1),
$$

$$
S(n+1) \triangleq H(n+1)P(n+1|j)H(n+1)^T + R_n(n+1),
$$

where $u = 1, \ldots, U$ and $R_n$ is the measurement noise covariance matrix. In the base/rover framework, $R_n$ is given by $R_n = \text{diag} \left[ \sigma^2_{\text{nav}_1}, \sigma^2_{\text{base}_1}, \ldots, \sigma^2_{\text{nav}_U}, \sigma^2_{\text{base}_U} \right]$; however, in the standalone framework, it is given by $R_n = \text{diag} \left[ \sigma^2_{\text{nav}_1}, \ldots, \sigma^2_{\text{nav}_U} \right]$. $H$ is the Jacobian matrix defined as

$$
H(n+1) = \begin{bmatrix} H^{(1)}(n+1) \\
\vdots \\
H^{(U)}(n+1) \end{bmatrix}.
$$

The estimation error covariance matrix is updated according to

$$
P(n+1|n+1) = [I - K(n+1)H]P(n+1|j).
$$

Note that the LTE navigator receiver’s position was assumed to be identical to the IMU’s position, i.e., $r_{\text{nav}} \equiv r$.

C. Framework Comparison

This subsection discusses the pros and cons of each framework. The base/rover framework has less clock states to estimate, i.e., 2 states. Besides, the base and rover clocks are characteristic offline, i.e., $Q_{\text{clk}}$ are known a priori. However, this framework requires a base and a communication link between the rover and the base. On the other hand, the standalone framework does not need a base or a communication link. However, It as more states to estimate, i.e., $2U$ and the clocks of eNodeBs are harder to characterize a priori.

VI. COMPUTATIONAL COMPLEXITY

The computational cost of the proposed system can be divided into 3 parts: (1) the LTE receiver, (2) the SAN beamforming process, and (3) the EKF navigation filter. The complexity of the proposed software-defined radio (SDR) is on the order of $O(\text{SDR}) \approx O(N_c \log N_c)$ [56]. In the beamforming process, the main computational cost is due to the DOA estimation technique, i.e., ESPRIT algorithm. Thus, the computational cost of the feedforward SAN algorithm is $O(\text{SAN}_{FF}) \approx O(\text{ESPRIT}) = O(N(L+1)M)$ [64]. The computational cost of the EKF is max$(O(s^{2.376}), O(g(n+1)), O(h(n+1)))$, where $s$ is the number of states, $g(n+1)$ is the dynamics model, and $h(n+1)$ is the measurement model [75], [76]. The proposed navigation framework assumes simple propagation and measurement models, which lead to a computational complexity of $O(s^{2.376})$. It is worth mentioning that in some studies in literature, the computational complexity of EKF is assumed to be $O(s^3)$. The difference is due to different algorithms used to solve for matrix inversion or multiplication. For this paper, the most efficient algorithm is considered, which gives a computational complexity of $O(s^{2.376})$. To this end, the number of states differ among the two proposed frameworks which leads to different computational cost: (i) the base/rover (B/R) framework with $O(\text{EKF}_{B/R}) = O((10)^{2.376})$ and (ii) the standalone rover (SR) framework with $O(\text{EKF}_{SR}) = O((8+2U)^{2.376})$. Therefore, the overall computational complexity of the system in the feedforward coupling scheme is approximated as

$$
O_{FF} \approx \begin{cases} 
O(\text{SDR}) + O(\text{SAN}_{FF}) + O(\text{EKF}_{B/R}), & \text{if B/R} \\
O(\text{SDR}) + O(\text{SAN}_{FF}) + O(\text{EKF}_{SR}), & \text{if SR}.
\end{cases}
$$

However, for the feedback coupling scheme, the complexity can be approximated as

$$
O_{FB} \approx \begin{cases} 
NO(\text{SDR}) + O(\text{SAN}_{FF}) + O(\text{EKF}_{B/R}), & \text{if B/R} \\
NO(\text{SDR}) + O(\text{SAN}_{FF}) + O(\text{EKF}_{SR}), & \text{if SR}.
\end{cases}
$$

VII. EXPERIMENTAL RESULTS

To evaluate the performance of the proposed framework, an experiment was conducted in an indoor environment: Winston Chung Hall building at the University of California, Riverside, USA. This section presents the experimental setup, analyzes the performance of the carrier phase-based receiver with and without SAN coupling, demonstrates the performance of the LTE-SAN-IMU system, and characterizes the performance of the SAN-based spatial discriminator.

A. Environmental Layout and Hardware

The base was placed on the roof of the building, while the rover was placed indoors. Both the base and rover receivers were equipped with four consumer-grade cellular omnidirectional antennas to collect LTE data at four different carrier frequencies. These frequencies corresponded to three U.S. LTE cellular providers whose characteristics are summarized in Table III. The base used three single-channel National Instruments (NI) universal software radio peripherals (USRPs)-2920 to simultaneously down-mix and synchronously sample LTE signals at 10 Msps. The signals were recorded on a laptop, which was connected to the USRPs through an ethernet cable. The base location was estimated using a GPS receiver.
The rover’s hardware setup was similar to the base except for the USRP configurations, which were a dual-channel USRP-2954R and two USRPs-2920. The USRPs at the rover simultaneously down-mixed and synchronously sampled LTE signals at 20 Mega-samples per second (Msps). The rover was equipped with a lower-end tactical-grade IMU (Vectornav VN-100) that outputs inertial measurements at a rate of 100 Hz. The signals were processed in a post-processing fashion. Although the SAN snapshots are taken at a higher resolution in time-domain, i.e., the duration of the OFDM symbol which is 66.7 μs. The navigation observables are obtained at the OFDM frame rate which is 100 Hz.

Several tags were placed at known locations on the ground before performing the experiment. Over the course of the experiment, a smart phone camera was used to record the location of the rover using the tags on the ground, which were later used as the ground truth. Fig. 9 shows the base and rover experimental hardware setup.

![Rover and base experimental hardware setup](image)

**Fig. 9.** (a) Rover and (b) base experimental hardware setup.

Fig. 10 shows the environmental layout of the experiment and the location of the eNodeBs to which the base and rover receivers were listening.

![Location of the LTE eNodeBs](image)

**Fig. 10.** The location of the LTE eNodeBs to which the base and rover receivers were listening and the environmental layout: Winston Chung Hall building at the University of California, Riverside, USA. Image: Google Earth.

### B. EKF Initialization and Settings

It was assumed that the rover entered the building from outside and that it had access to GPS signals at  \( k = 0 \) and \( k = 1 \). This allows the rover to estimate its position. The receiver’s clock bias \( c \Delta \hat{t} \) and drift \( c \Delta \dot{t} \) were initialized using the receiver’s initial position and two consecutive prior measurements. The initial clock bias and drift uncertainties were set to \( 1 \) m² and 0.1 (m/s)², respectively. It was assumed that the receiver was equipped with a temperature-compensated crystal oscillator (TCXO) and values of \( S_{\delta t, x} \) and \( S_{\delta t, y} \) were set to \( 4.7 \times 10^{-20} \) and \( 7.5 \times 10^{-20} \), respectively [57].

The measurement noise variance \( \{ \sigma_i^2, \sigma_u^2 \}_i \) for \( i \in \{ \text{nav}, \text{base} \} \) were set to \( \{ \sigma_i^2, \sigma_u^2 \}_i \) respectively, where \( \{ C/N_0 \}_i \) is the received carrier-to-noise ratio for the \( i \)-th eNodeB and \( \{ n \}_i \) are tuning parameters that were chosen to be \( \{ 5.56, 7.78, 3.33, 3.1, 3.78 \} \times 10^{-12} \).

The rover’s initial position and orientation were considered as the origin and orientation, respectively, of the local frame in which the rover’s motion state was estimated. The gyroscope’s and accelerometer’s biases were initialized by taking the mean of 30 seconds of IMU data, while the rover was stationary. The rover’s initial orientation, position, and velocity were initialized using a multivariate Gaussian random generator with a mean \( \mathbb{E} \left\{ \hat{\theta}_x(0|0), \hat{r}^T(0|0), \hat{r}^T(0|0) \right\} = [0, 0, 0, 2.2, 0.2] \) and a covariance of \( \mathbf{P}(0|0) = \text{diag}[0.1, 9, 9, 1, 1] \).

### C. Navigation Solution

The performance of the proposed navigation frameworks are evaluated in this subsection. Over the course of experiment, the rover traversed a trajectory of 109 m, while the base was stationary. Fig. 11 shows the tracking results of the rover: (a) the estimated and actual carrier phases (in meters) for each eNodeB, (b) the obtained errors after removing the initial error, and (c) the measured \( C/N_0 \) of the received signal from each eNodeB over the entire experiment. The \( C/N_0 \) results are consistent with results in [20], where the \( C/N_0 \) of LTE signals from all eNodeBs was powerful over the entire indoor experiment.

Referring to Subsection III-D, it is worth mentioning that the dynamics of \( \delta t_i \) and \( \delta \dot{t}_i \) are unstable; hence, \( \phi_{i, u}(n) \) is an increasing sequence as shown in Fig. 11. Yet, the receiver is still capable of tracking the signal unless the drift of the clock is outside the dynamic range of the tracking loops, which is unlikely given the qualities of the deployed clocks. For example, Fig. 12 shows the carrier phase error produced by the receiver of eNodeB 1 throughout the entire experiment along with the \( \pm 15^\circ \) bounds, where bounded carrier phase errors were maintained. However, the clock errors should be accounted for in the navigation filter in order to produce an accurate navigation solution as discussed in Section V. The same behavior is observed for other eNodeBs.

Different navigation frameworks were compared with each other’s and with respect to the ground truth, namely: (1) IMU only, (2) LTE, (3) feedforward LTE-SAN, (4) feedback LTE-SAN, (5) LTE-IMU, and (6) LTE-SAN-IMU with feedback LTE-SAN. Note that the navigation solution corresponding to (2)-(6) were obtained via (i) base/rover (Fig. 13(a)) and (ii) standalone rover framework (Fig. 13(b)). Table IV summarizes the experimental results. It is worth noting that (i) slightly outperforms (ii), but this comes at the expense of needing a base, which may not be feasible in some applications.

Fig. 14 shows the EKF estimation error of the navigator’s \( x \)-position and \( y \)-position along with the associated \( \pm 2 \sigma \) bounds.

### D. SAN-Based Beamforming Results

This subsection analyzes the SAN-based beamforming process specifically: (i) LOS DOA RMSE of the standard ESPRIT
algorithm and (ii) effect of the size of the synthetic antenna array on the localization accuracy. Fig. 15 shows the LOS DOA RMSE of the standard ESPRIT algorithm for different LTE eNodeBs over the entire experiment. In light of these results, the accuracy of the LOS DOA estimates requires to beamform using a relatively wide beam to guarantee capturing the LOS component; however, this trade-off between capturing LOS component and suppressing multipath components may introduce more multipath-induced errors.

In the proposed framework, the size of the synthetic antenna array is a significant parameter that is related to the stationarity of the wireless channel, as discussed in Subsection III-F. For the performed experiment, a study was conducted to show the effect of the size of the synthetic antenna array on the localization accuracy of the proposed system. Fig. 16 shows the effect of $N$ on the position RMSE of the FB-LTE-SAN for $N = \{2, 3, \ldots, 16\}$. In light of these results, three regions can be identified as follows:

![Diagram](image-url)
not among the most powerful. For \( N \geq 9 \) shaded in red.

The yellow region shows a reduction in position RMSE as \( N \) increases from 2 to 7. For \( N = 8 \), the best performance was achieved with a position RMSE of 4.05 m. This is due to increasing the DOF of the proposed system, i.e., as \( N \) increases the proposed SAN-based beamforming process can capture more incoming signals, and consequently increase the possibility of capturing the LOS signal. In other words, the ESPRIT will estimate the DOA of the most powerful \( N - 1 \) signals in the case where \( N - 1 \geq L + 1 \). If the LOS signal is not among the most powerful \( N - 1 \) signals, the DOA estimates represent multipath DOA estimates and the system fails to capture the LOS components.

However, for \( N \geq 9 \), the red region shows a significant increase in the position RMSE. This can be justified due to the stationarity of the LTE wireless channel. In this case, the assumption of having time-separated snapshots as geometrically-spaced antenna elements does not hold anymore due to channel variations. This introduces significant error in the SAN-based beamforming process by producing faulty DOA estimates, and consequently corrupting the beamformed data.

To justify this result, a theoretical approximation can be found from equations (7)-(9). For this purpose, (7) and (8) can be expanded as

\[
T_c = \frac{0.432c}{v_{\text{max}} f_c}, \\
F = \left[ \frac{\alpha_{\text{min}} c}{t_{\text{symb}} v_c f_c} \right],
\]

where \( f_c \) is the carrier frequency of the received LTE signal. Then, (9) can be written as

\[
N_{\text{max}} = \left[ \frac{0.432c}{v_{\text{max}} f_c t_{\text{symb}}} \right],
\]

where the approximation from (22) to (23) is due to fact that \( \frac{\alpha_{\text{min}} c}{t_{\text{symb}} v_c f_c} \gg 1 \) for \( \alpha_{\text{min}} = 0.05 \) and \( f_c \in [600 \text{ } 3000] \text{ MHz} \) (i.e., range of LTE frequencies). In this experiment, \( v_c \) on average was constant throughout the experiment; thus, \( \frac{v_c}{v_{\text{max}}} \approx 1 \), which results in \( N_{\text{max}} \approx 8 \).

VIII. CONCLUSION

This paper presented an infrastructure-free, practical, affordable, and accurate indoor navigation and localization system using downlink LTE signals and an IMU. The proposed system exploits the motion of a single antenna element to spatially discriminate LOS from multipath signals in an SAN framework, and subsequently, beamforms the incoming signals towards the LOS direction while minimizing the multipath components. The paper discussed the carrier phase-based LTE receiver to extract the navigation observables and presented two navigation frameworks to unknown clock biases of LTE eNodeBs: (i) base/rover and (ii) standalone rover. The different stages of the SAN-based beamforming process was discussed: (i) data formulation, (ii) preprocess filtering, (iii) model order estimation, (iv) DOA estimation, and (v) multipath mitigation. An EKF-based tightly-coupled LTE-SAN-aided IMU system was developed and the computational complexity of the proposed system was studied. Experimental results were presented, in which a pedestrian-mounted receiver navigated indoors for 109 m in 50 seconds, while receiving LTE signals from 5 LTE eNodeBs. Six navigation approaches were compared: (i) IMU only, (ii) LTE only, (iii) feedforward LTE-SAN, (iv) feedback LTE-SAN, (v) LTE-IMU, and (vi) feedback LTE-SAN-IMU. The position RMSE resulting from these approaches were 9.48 m, 5.09 m, 4.95 m, 4.05 m, 2.92 m, and 1.44 m, respectively.

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