Optimal GPS Integrity-Constrained Path Planning for Ground Vehicles

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Abstract—Path planning for a ground vehicle in an urban environment is considered. The vehicle is equipped with a GPS receiver and a road map. The vehicle desires to take the shortest path to reach a target destination, while guaranteeing that integrity monitoring-based measures are satisfied along its traversed path. A path planning algorithm is proposed that yields the optimal path to follow as well as suboptimal feasible paths. The integrity monitoring-based measure considered in this paper is the horizontal protection level (HPL), which refers to the statistical bound around the vehicle that guarantees the probability of the absolute position error exceeding a desired threshold is not larger than the integrity risk. Experimental results are presented showing that choosing the optimal path from the proposed algorithm reduces the average and maximum HPL by 2 m and 20.2 m, respectively, compared to choosing the shortest-time path, while introducing a negligible additional path length.

Index Terms—Navigation, ground vehicle, integrity, receiver autonomous integrity monitoring, path planning, GPS.

I. INTRODUCTION

The U.S. National Highway Traffic Safety Administration (NHTSA) reported that in 2018, there were more than 36,000 traffic fatalities, 1.8 million injuries, and 6.7 million crashes in the U.S. [1], with about 94% of crashes being attributed to human error. Autonomous ground vehicles have the potential to dramatically reduce vehicle collisions due to human error by reducing the number of humans behind the driving wheel. For reliable autonomous or semi-autonomous driving, the vehicle needs to be equipped with extremely reliable and accurate local and global sensing modalities to sense the surrounding environment and localize the vehicle within a global map. This cannot be achieved without continuously monitoring the integrity of the navigation solution provided by the vehicle’s navigation system. Integrity monitoring refers to the ability of the navigation system to provide timely warnings when the information given by its on-board sensors is not trustworthy. A high-integrity navigation system must be able to detect and reject incorrect measurements [2].

The concept of integrity was first formalized in the field of aviation, which is highly dependent on global navigation satellite systems (GNSS) [3]. Integrity monitoring can be established through the GNSS navigation message to indicate satellite anomalies, such as clock errors. However, this type of integrity monitoring is not practical for real-time applications, as it takes a few hours to identify and broadcast satellite failure. As such, alternative methods have been developed for real-time integrity monitoring. These methods can be categorized into internal and external. On one hand, integrity can be established using external methods, such as ground-based augmentation systems (GBAS) or satellite-based augmentation systems (SBAS) using reference receivers. On the other hand, integrity can be established using internal methods, such as receiver autonomous integrity monitoring (RAIM) using redundant information present in the measurements. RAIM inherently possesses desirable characteristics for ground-based receivers, and its design flexibility is well suited for navigation in urban environments. Moreover, RAIM can be used to generate the integrity monitoring measures, such as protection levels (PLs), which are statistical error bounds computed so as to guarantee that the probability of the absolute position error exceeding a certain threshold is smaller than or equal to a target integrity risk [4].

GNSS satellite visibility degrades in deep urban canyons and the received signals suffer from multipath and non-line-of-sight (NLOS) conditions. To overcome the limitations of GNSS, fusing GNSS receivers with other sensors [5]–[10] and signals of opportunity (SOPs) [11]–[16] is commonplace. Integrity monitoring for multi-sensor and multi-signal navigation systems has been the subject of numerous studies. This includes the fusion of (i) GPS, inertial measurement units (IMUs), wheel speed encoders, and cameras [17]; (ii) GPS and map-matching [18], [19]; (iii) GPS and lidar [20]; (iv) GNSS and IMU [21]; (v) GNSS, lidar, and IMU [22]; (vi) SOPs and GNSS [23], [24]; and (vii) SOPs [25], [26].

Path planning has been considered in recent literature to account for various sources of uncertainty (e.g., environmental [27]–[29], sensing [30], [31], etc.). Predicting GNSS signal power and availability was proposed in [32]–[34]. Such predictions could be useful for path planning purposes. The objective of path planning is to optimize a cost function, such as path length or path duration between a start and a target destination. Path planning to optimize the path length, while taking into account the accuracy of the vehicle’s estimated position from
GNSS and SOPs has been considered [35]–[38]. However, to the author’s knowledge, path planning in the context of integrity monitoring has not been studied yet. To this end, this paper considers path planning to minimize the vehicle’s path length, while upper bounding the vehicle’s horizontal protection level (HPL) to be less than a pre-defined threshold, which is known as the horizontal alert limit (HAL). As such, the proposed framework assures safe operation by prescribing the protection level (HPL) to be less than a pre-defined threshold, path length, while upper bounding the vehicle’s horizontal protection level (HPL) to be less than a pre-defined threshold, which is known as the horizontal alert limit (HAL). As such, the proposed framework assures safe operation by prescribing the protection level (HPL) to be less than a pre-defined threshold, which is known as the horizontal alert limit (HAL). 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where $\mathbf{B} \triangleq (\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H})^{-1} \mathbf{H}^T \mathbf{R}^{-1}$, $\mathbf{S} \triangleq \mathbf{I} - \mathbf{HB}$, and $X_{i,j}$ denotes the element of $i$-th row and $j$-th column of a matrix $\mathbf{X}$.

The slope is usually computed for each satellite individually, denoted by $s_{\text{gps}}$ for the $i$-th GPS satellite. When the slopes are large, the position error becomes more sensitive to the error in the test statistic, making the RAIM system less likely to detect a fault. Therefore, an important quantity to study is the maximum slope, denoted $s_{\text{max}}$, to which the HPL is proportional, i.e.,

$$ HPL = s_{\text{max}} \sqrt{\lambda_{\text{det}}} $$

where $\lambda_{\text{det}}$ is the non-centrality of the test statistic chi-squared distribution under a faulty operation that results in a predefined probability of missed detection $P_{\text{MD}}$, according to

$$ P_{\text{MD}} = \int_0^{T_h} f_{X_{\text{MD}},\lambda_{\text{det}}} (\tau) d\tau, $$

where $f_{X_{\text{MD}},\lambda_{\text{det}}}$ represents the non-central chi-squared pdf with $d$ degrees of freedom and non-centrality parameter $\lambda_{\text{det}}$, and $T_h$ is the threshold for the test statistic and is obtained from

$$ P_{\text{FA}} = \int_{T_h}^{\infty} f_{X_{\text{MD}},\lambda_{\text{det}}} (\tau) d\tau, $$

where $P_{\text{FA}}$ is the probability of false alarm and $f_{X_{\text{MD}},\lambda_{\text{det}}}$ is the test statistic chi-squared pdf under fault-free condition [26].

### III. PATH PLANNING

#### A. Road Map Generation

To extract road data for path planning, a digital map has been used, which covers the city of Irvine, California, USA, and is developed based on the Open Street Map (OSM) database [41]. OSM is built by a community of mappers that contribute and maintain roads, trails, and railway stations information. A MATLAB-based parser was developed to extract the road coordinates. The coordinates of the road were obtained using Google Earth [42]. The MATLAB-based parser outputted the coordinates of the nodes (i.e., the street junctions), the roads, and the connectivity matrix $\Theta$, which is defined according to

$$ \Theta_{i,j} = \begin{cases} 
1 & \text{if there exists a road between nodes } i \text{ and } j \\
0 & \text{otherwise}
\end{cases} $$

Fig. 1 summarizes the steps to extract path planning related data from a digital map. Fig. 1(a) shows the navigation environment. Fig. 1(b) demonstrates the same area in OSM database, which is downloadable from the OSM website [41]. Fig. 1(c)–(e) show the steps to process the map data and to extract the nodes, roads, and the connectivity matrix. Fig. 1(f) shows the extracted roads and road overlaid on the map. This area contains 8,368 nodes and 1,472 roads, which are illustrated with red circles and blue lines, respectively. An example of the path between two sample points is depicted in Fig. 1(f). This path, $\pi$, contains 8 nodes, labeled as

$$ \pi = \{464, 236, 112, 234, 30, 31, 114, 118\}. $$

Finally, Fig. 1(g) illustrates the connectivity between different nodes.

#### B. Path Planning Engine

The path planning generation step prescribes an optimal path for the vehicle to follow. This subsection describes the steps to determine the optimal path between a start and target nodes on the digital map. The optimal path is one that accounts for the shortest path length and for the HPL. To account for both HPL and path length, the optimization cost function is chosen to be the sum of the HPL along the path, multiplied by the distance between two adjacent nodes. The distance is explicitly considered in the cost function because only including HPL could result in lengthy paths, e.g., paths that require the vehicle to leave and re-enter the urban environment. Fig. 2 summarizes the block diagram of the path planning engine with the corresponding inputs and outputs.

Formally, a path from the start to the target nodes is denoted $\pi \in \mathcal{P}$, where $\mathcal{P}$ is the set of all paths. The path $\pi$ is composed of a sequence of nodes indices between the start node index $p_s$ and the target $p_g$, namely $\pi = \{p_s, p_1, p_2, \ldots, p_g\}$. The optimization problem is expressed as

$$ \begin{aligned} 
\text{minimize} & \sum_{p \in \pi} \text{dist}(p) \cdot HPL(p, t) \\
\text{subject to} & HPL(p, t) \leq \text{HAL}
\end{aligned} $$

where $\text{dist}(p)$ is the length of the road network between node $p$ and its adjacent node and $HPL(p, t)$ is the predicted $HPL$ in node $p$ at time $t$. 

![Fig. 1. Steps to extract path planning related data from a digital map: (a) The navigation environment, (b) OSM digital map, (c) exporting the “.osm” file, which contains road data, (d) MATLAB-based parser to extract nodes and roads data from the “.osm” file, (e) processing the digital map, including constructing the connectivity matrix, (f) the extracted nodes and road overlaid on the map, and (g) the connectivity matrix.](image)
The optimization problem in (3) resembles the problem of finding the shortest path in a weighted graph, in which the roads are the edges of the graph and the cost function

\[ f(\beta, \alpha) = \sum_{p \in P(\beta, \alpha)} \text{dist}(p) \cdot HPL(p, t), \]

(4)
determines the weight of the edge that connects the node \( \alpha \) to \( \beta \). Based on the constraints in (3), if \( HPL(p, t) \) exceeds HAL for \( p \in P(\beta, \alpha) \), then the edge is removed from the graph. To solve the optimization problem expressed in (3), Dijkstra’s algorithm is employed, which is a classic algorithm to find the shortest path between two arbitrary nodes of a weighted graph. Dijkstra’s algorithm is readily implementable, performs stably, and has acceptable complexity. The proposed algorithm is implemented as follows. Assume that the vehicle is driving in a region consisting of \( n \) nodes and \( w \) roads. This region can be modeled by a simple graph \( \mathcal{G} = (n, w) \), which consists of \( n \) nodes and \( w \) edges. The path planning cost function \( f(\beta, \alpha) \) assigns a non-negative real number weight to the edge from \( \beta \) to the \( \alpha \) in \( \mathcal{G} \). Define \( s \) to be the start node from which the vehicle begins driving, \( g \) to be the target node, and \( d(g) \) to be the weight related to the path from \( s \) to \( g \). Let \( S \) denote the set of visited nodes by the vehicle and \( V \) denote the set of unvisited nodes. Given a path \( \pi \) in \( \mathcal{G} \) determined by the algorithm, \( \alpha_p \) denotes the predecessor of node \( \alpha \). The path planning is initialized as follows:

- \( d(s) = 0 \)
- For each node \( \alpha \) adjacent to \( s \), set \( d(\alpha) = f(s, \alpha) \) and \( \alpha_p = s \)
- For each node \( \alpha \) such that \( \alpha \neq s \) and \( \alpha \) is not adjacent to \( s \), set \( d(\alpha) = \infty \)
- \( S = \{ s \} \)

Next, the path planning algorithm executes the steps outlined in Algorithm 1.

IV. EXPERIMENTAL RESULTS

This section evaluates the efficacy of the proposed path planning algorithm experimentally on a ground vehicle.

![Fig. 2. The block diagram of path planning engine with the corresponding inputs and outputs.](image)

![Fig. 3. Experimental setup. Vehicle used to conduct the experiment, which was equipped with the AsteRx-i V® GNSS-IMU module and a laptop for storage and processing.](image)

**Algorithm 1: Path Planning Algorithm**

**Input:** \( \mathcal{G}, s, g, S, \) and \( f(\beta, \alpha) \)

**Output:** \( d(g) \) and \( \pi(g) \)

1. Find \( \alpha \in V \) that minimizes \( d(\alpha) \)
2. For each \( \beta \) adjacent to \( \alpha \)
3. If \( d(\alpha) + f(\beta, \alpha) < d(\beta) \),
   \[ d(\beta) = d(\alpha) + f(\beta, \alpha) \]
4. \( \beta_p = \alpha \)
5. End if
6. End for
7. \( V \leftarrow V - \{ p \} \)
8. \( S \leftarrow S + \{ p \} \)
9. If \( S \neq V \),
10. Goto Step 1
11. End if

**A. Experimental Setup and Scenario Description**

A vehicle was equipped with a Septentrio AsteRx-i V® integrated GNSS-IMU module, which is equipped with a dual antenna, multi-frequency GNSS receiver and a Vectornav VN-100 micro-electromechanical system (MEMS) IMU. This integrated GNSS-IMU system was used to produce the vehicle’s ground truth path. The GNSS receiver also produced GPS pseudorange measurements, which were used to construct the HPL as discussed in Subsection II-B. The experimental setup is shown in Fig. 3.
settings were $P_{MD} = P_{FA} = 0.005$ and $HAL = 70$ m. The experiment considered four path between the start and target points: optimal path, suboptimal feasible path, and two infeasible paths. The HPL was computed from experimental data and compared for all paths. To account for GPS satellite motion, a MATLAB-based parser was developed to generate satellite positions from online Receiver Independent Exchange (RINEX) files. For a fair comparison, four driving campaigns took place over four successive days: January 22–25, 2020. The start time of the tests were chosen to be 9:23am, 9:19am, 9:15am, and 9:11am to guarantee the same satellite configuration at the beginning of each test [32]. The vehicle used GPS satellites to which there was a clear LOS, with the number of used GPS satellites along the paths varying between 5 and 13.

B. Experimental Results

Fig. 4 shows the four paths along with the time spent traversing each path. The path length, average HPL, and maximum HPL for each path are tabulated in Table I. The proposed algorithm returned Path 2 as the optimal path and Path 4 as a suboptimal feasible path. Paths 1 and 3 were returned as infeasible paths, but were traversed in the experiment for comparison purposes, since Path 1 was chosen by the ubiquitous navigation software Google Maps as the shortest-time path to follow, while Path 3 appeared attractive as it produced the lowest average HPL.

The following may be concluded from these results:

- The path lengths and traversed time of Path 1 and Path 2 were comparable. However, the HPL along Path 1 violated the desired HAL and was returned as infeasible.
- The smallest average HPL was experienced in Path 3. However, the HPL along Path 1 violated the desired HAL and was returned as infeasible.
- Choosing the optimal path prescribed by the proposed framework reduced the average and maximum HPL by 2 m and 20.2 m, respectively, compared to choosing the shortest-time path proposed by Google Maps.

<table>
<thead>
<tr>
<th>Path</th>
<th>Length [m]</th>
<th>Traverse time [minutes]</th>
<th>Average HPL [m]</th>
<th>Maximum HPL [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path 1 (infeasible, shortest time)</td>
<td>9746</td>
<td>15</td>
<td>11.9</td>
<td>82.1</td>
</tr>
<tr>
<td>Path 2 (optimal)</td>
<td>9631</td>
<td>17</td>
<td>9.9</td>
<td>61.9</td>
</tr>
<tr>
<td>Path 3 (infeasible)</td>
<td>14244</td>
<td>26</td>
<td>9.6</td>
<td>92.1</td>
</tr>
<tr>
<td>Path 4 (suboptimal feasible)</td>
<td>10629</td>
<td>16</td>
<td>9.7</td>
<td>69.1</td>
</tr>
</tbody>
</table>

V. Conclusion

This paper proposed a framework for optimal integrity-constrained ground vehicle path planning. The framework prescribed the optimal path between start and target points, while accounting for both HPL and path length and guaranteeing HPL being upper bounded by a desired HAL. Experimental results were presented to validate the efficacy of the proposed framework, showing that choosing the optimal path reduced the average and maximum HPL by 2 m and 20.2 m, respectively, compared to choosing the shortest-time path, while introducing a negligible additional path length.

VI. Acknowledgment

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